

# FactsWise: Developing Addition and Subtraction Fluency and Flexibility

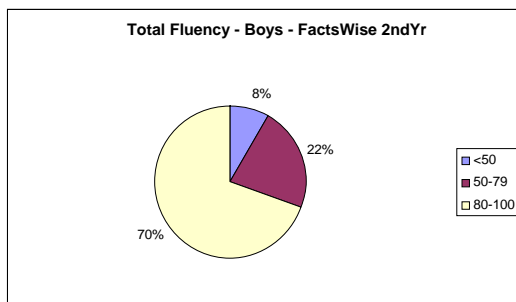
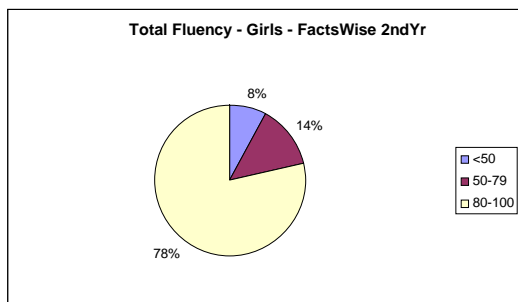
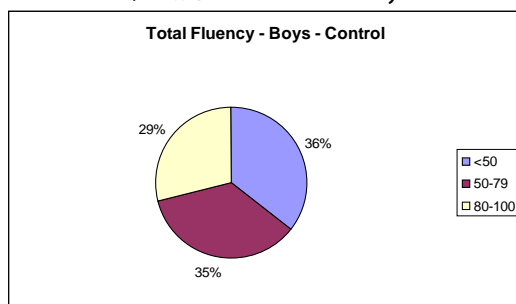
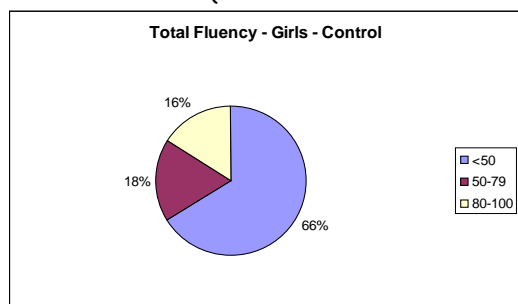
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FactsWise breaks the facts up into 9 small chunks, with an early focus on 5s and 10s. Consistent with the ways students from many other countries are taught their facts, students are then taught part-whole strategies using 5s and 10s to solve larger facts, including:

$$\begin{array}{c} 8 + 5 \\ \swarrow \quad \searrow \\ 8 + 2 + 3 \\ \swarrow \quad \searrow \\ 10 + 3 = 13 \end{array} \quad \text{or} \quad \begin{array}{c} 15 - 8 \\ \swarrow \quad \searrow \\ 5 + 10 - 8 \\ \swarrow \quad \searrow \\ 5 + 2 = 7 \end{array} \quad \text{or} \quad \begin{array}{r} 15 \\ - 8 \\ \hline \end{array} \rightarrow \begin{array}{r} 10 + 5 \\ - 8 \\ \hline 2 + 5 = 7 \end{array}$$

## Percent of Students Demonstrating Basic Facts Fluency

Solving 10 addition and 8 subtraction facts without counting in 3 seconds or less  
(98 control students and 141 FactsWise students from Decile 10 Schools)



Yellow - 80-100% fluent; Red - 50-79% fluent; Blue - less than 50% fluent

## RESEARCH-BASED PRINCIPLES FOR BUILDING BASIC FACTS FLUENCY

1. Focus on accuracy, fluency, and part-whole thinking
  - Encourage students to move from counting to strategies as well as memorization
2. Teach new material in a hierarchical order
3. Teach subtraction/division facts just after related addition/multiplication facts
4. Ongoing whole-class instruction & practice (every 24-48 hours)
  - Elaborative Rehearsal
  - Immediate Feedback
5. Ongoing one-on-one assessment
  - Continuing press to move beyond counting
6. Systematic homework

## NINE GOALS FOR BASIC FACTS SUCCESS

<b>Goal 1 - Within 4s &amp; 5s</b> 1+3, 2+2, 3+1, 1+4, 2+3, 3+2, 4+1 4-1, 4-2, 4-3, 5-1, 5-2, 5-3, 5-4	<b>Goal 5 - With 5s (part 2)</b> 5+6, 5+7, 5+8, 5+9 11-5, 11-6, 12-5, 12-7, 13-5, 13-8, 14-5, 14-9
<b>Goal 2 - With 5s (part 1)</b> 1+5, 2+5, 3+5, 4+5, 5+5 6-1, 6-5, 7-2, 7-5, 8-3, 8-5, 9-4, 9-5, 10-5	<b>Goal 6 - Doubles</b> 3+3, 4+4, 6+6, 7+7, 8+8, 9+9 6-3, 8-4, 12-6, 14-7, 16-8, 18-9
<b>Goal 3 - Within 10s</b> 0+10, 1+9, 2+8, 3+7, 4+6 10-0, 10-10, 10-1, 10-9, 10-2, 10-8, 10-3, 10-7, 10-4, 10-6	<b>Goal 7 - Under Tens</b> 2+4, 2+6, 2+7, 3+4, 3+6 6-2, 6-4, 8-2, 8-6, 9-2, 9-7, 7-3, 7-4, 9-3, 9-6
<b>Goal 4 - With 10s</b> 10+1, 10+2, ... 10+9, 10+10 11-1, 11-10, ... 18-8, 18-10, 19-9, 19-10, 20-10	<b>Goal 8 - With 9s</b> 2+9, 3+9, 4+9, 6+9, 7+9, 8+9 11-2, 11-9, 12-3, 12-9, 13-4, 13-9, 15-6, 15-9, 16-7, 16-9, 17-8, 17-9
<b>Goal 9 - With 7s &amp; 8s</b> 4+7, 6+7, 3+8, 4+8, 6+8, 7+8 11-4, 11-7, 13-6, 13-7, 11-3, 11-8, 12-4, 12-8, 14-6, 14-8, 15-7, 15-8	

### Correlating Common Core State Standards and FactsWise Goals

FactsWise Goals	Common Core State Standards Expectations			
	Kindergarten	1st Grade	2nd Grade	By end of 2nd Grade
1: Within 4s & 5s (1+3, 2+2, ...)	F + -	F + -	F + -	M + -
2: With 5s (5+1 to 5+5)	D + -	F + -	F + -	M + -
3: Within 10s (1+9, 2+8, ...)	D + -	F + -	F + -	M + -
4: With 10s (10+1 to 10+10)	D +	F + -	F + -	M + -
5: With 5s (5+6 to 5+9)		S + -	F + -	M + -
6: Doubles		S + -	F + -	M + -
7: Misc. Under 10s (2+6, 3+7, ...)	D + -	F + -	F + -	M + -
8: With 9s (2+9 to 8+9)		S + -	F + -	M + -
9: With 7s & 8s (4+7, 6+7, ...)		S + -	F + -	M + -

- D - Developing: Solve by using objects or drawings to represent the problem
- F - Fluently add and subtract using mental strategies
- M - Know from memory
- S - Use strategies such as decomposing, making ten, creating equivalent but easier or known sums, ...

## Common Core State Standards related to Addition and Subtraction Facts Fluency

### Kindergarten:

- Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
- Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g.,  $5 = 2 + 3$  and  $5 = 4 + 1$ ).
- For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
- Fluently add and subtract within 5.
- Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g.,  $18 = 10 + 8$ ); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

### 1st Grade:

- Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g.,  $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$ ); decomposing a number leading to a ten (e.g.,  $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$ ); using the relationship between addition and subtraction (e.g., knowing that  $8 + 4 = 12$ , one knows  $12 - 8 = 4$ ); and creating equivalent but easier or known sums (e.g., adding  $6 + 7$  by creating the known equivalent  $6 + 6 + 1 = 12 + 1 = 13$ ).
- The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
- Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
- Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

### 2nd Grade:

- Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

## Whole-Class Routines - Concrete to Representational to Abstract

Goal 1	Goal 2	Goal 3	Goal 4
<b>Show Me</b> - have students, on one hand, show 1, 2, 3, 4, 5. Do this routine for 1 or 2 minutes per day until they are able to do this automatically.	<b>6, 7, 8, 9, 10</b> (have students show 6 as 5 & 1, 7 as 5 & 2, ...)		<b>Four Hand Show Me</b> For 16, one student shows 10 and the other student shows 6
<b>Slavonic Abacus Show Me</b> - some students show 1, 2, 3, 4, 5 on the Slavonic abacus; others show on one hand.	<b>6, 7, 8, 9, 10</b>		<b>11, 12, 13, ..., 20</b>
<b>All the Ways</b> - have 4 students stand in front of the class on the right side of the room - this represents $4=0+4$ . Have one student at a time walk to the left side of the room. Have the students in the class choral chant $4=1+3$ , $4=2+2$ , $4=3+1$ , $4=4+0$ . Use this same procedure for ways to make 5 (Goal 1 continued) and all the ways to make 10 (Goal 3).		<b>All the ways to make 10</b> Help students notice patterns and encourage them to develop a systematic approach (e.g., $0+10$ , $1+9$ , $2+8$ , $3+7$ , ...). Have students write and/or say all the ways to make 10 in three ways*:	
<b>One Hand Choral</b> - have students use <u>one hand</u> to model addition problems first ( $1+3$ makes, $2+3$ makes, ...). As students become fluent with addition facts, have students use one hand to model subtraction problems ( $4-1$ makes, $4-3$ makes, $5-2$ makes, ...).	<b>Two Hand Choral</b> - have students use two hands to model addition problems first ( $5+1$ makes, $5+2$ makes, ...). Also do subtraction ( $6-1$ makes, $7-5$ makes, ...).	<b>Two Hand Choral</b> - have students use two hands to model all the ways to make 10 ( $1+9$ , $2+8$ , ...). Also do subtraction ( $10-1$ is 9, $10-2$ is 8, ...).	<b>Four Hand Choral</b> - have pairs of students use four hands to model addition problems ( $10+8$ makes, ...). Also subtraction ( $16-6$ makes, $16-10$ makes, ...).
<b>Snap Cube Student Leaders</b> - have one or more students use snap cubes (unifix, multilink, ...) to model ways to make 4 and 5 and lead students in choral responses such as "4+1 makes" and "5-4 makes".	... "with 5s" (" $5+3$ makes, ..., " $8-5$ makes", ...)	... ways to make 10	... "with 10s" addition and subtraction

\* 1) Using equations such as  $10=0+10$ ,  $10=1+9$ , ...

2) In part-whole grids

3) In a table (horizontal and/or vertical)

10	
0	10

10	
1	9

0	1	2	3	4	5	6	7	8	9	10
10	9	8	7	6	5	4	3	2	1	0

Goal 1	Goal 2	Goal 3	Goal 4
<b>Ten Frame Choral</b> - show students ways to make 4 and 5 on ten-frame cards and use choral response - "1+3 makes".	... "with 5s" ("5+3 makes, ..., "8-5 makes", ...)	... ways to make 10	Use two Ten Frames "with 10s" addition and subtraction
<b>Part-Whole Grid Choral</b> - show students a part-whole grid (see FactsWise Part-Whole Resources) for a way to make 4 or 5 and have students state both the addition and subtraction relationships while you write them on the board.	... "with 5s" ("5+3 makes, ..., "8-5 makes", ...)	... ways to make 10	... "with 10s" addition and subtraction
<p><b>Arrow Cards</b> - show students a 10 arrow card and any 1-digit arrow card. Have students respond chorally as you slide the two arrow cards together. Have students respond in: 1) "place value order" (e.g., 10+8 makes "ten eight") and 2) standard number language (e.g., 10+8 makes eighteen).</p>			
<p><b>Hundred Chart Choral</b> - use the first two rows of a hundreds chart:</p> <ul style="list-style-type: none"> <li>* Point to a single-digit number such as 8 and ask students to add 10. Move your finger down to 18.</li> <li>* Point to a double-digit number such as 15 and ask students to subtract 10. Move your finger up to 5.</li> <li>* Point to a double-digit number such as 15 and ask students to subtract 5. Slide your finger to 10.</li> </ul>			
<b>Snap Facts</b> - Say a fact such as "2+3 makes" and then use a hand signal for wait. Give students 2 or 3 seconds to process, and then snap your fingers, at which time students will choral respond with the answer. Watch carefully for facts where some students are delaying their answers, and repeat those several times before you end the activity. Remember to spend more time on subtraction facts than you do on addition facts!	... "with 5s" ("5+3 makes, ..., "8-5 makes", ...)	... ways to make 10	... "with 10s" addition and subtraction

## Systematic, Goal-Specific Practice with Immediate Feedback

Goal 1	Goal 2	Goal 3	Goal 4
Pairs Practice "Tents"	Pairs Practice "Tents"	Pairs Practice "Tents"	Pairs Practice "Tents"
Subtraction War	Subtraction War	Go to the Dump card game	Arrow Card Game

Go to <https://sites.google.com/site/factswise/> - Addition Subtraction Resources for many of these resources. To find the Pairs Practice Tents, download the FactsWise Resource Book under Addition and Subtraction Resources.

**Goal 3 Game: Go to the Dump** (adapted from Math Card Games at [www.alabacus.com](http://www.alabacus.com))

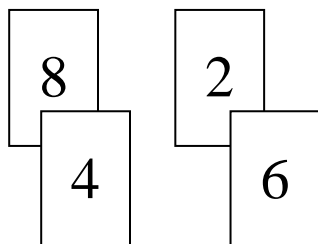
**Math Skills:** Know the addition facts of 10

**Materials:** 0 through 10 cards (or aces through 9 from a standard deck of cards)

**Source:** Adapted from [www.nzmaths.co.nz](http://www.nzmaths.co.nz)

**Directions:**

1. This activity is based on the card game Go Fish and is played in groups of 2 to 4.
2. Shuffle and then deal out all the cards to the players.
3. The children look through their cards and make pairs of cards that sum to 10. These pairs are placed front up in front of each child.
4. The children take turns asking one another for a card that they can use to make 10 with a card in their hand.
5. If the child asked has the card they must give it up. If not, the child says "Go to the Dump."
6. If the turn-taking child makes a pair, he/she places it on the table. His/her turn is over whether or not a pair is made.
7. The aim of the activity is to get the most pairs that sum to 10.



## Goal 4: The Arrow-Card Game

### Math Skills:

- Developing the addition and subtraction facts for +10 and -10

### Materials:

- One set of arrow cards for each student (2 or more students can play)
- Hundred Number Chart and Markers (1 per student)

### Directions:

1. Each student places the arrow card for 10 in front of him or her. The rest of the arrow cards are placed face down in a draw pile in the center of the table.
2. On each turn, the student draws an arrow card from the draw pile.
3. The student makes a decision of whether to add the draw card to the 10, or subtract it from the 10. The student then moves her/his marker on the Hundred Number Chart the number of spaces of the sum or difference.
4. The game is over when the draw pile is empty.
5. The student who is closest to the number 100 wins.

## **Goals 5 through 9: Part-Whole Thinking and the Power of Tens**

There are three main kinds of part-whole strategies for addition: making tens, working with fives, and relating to known facts.

Many U.S. teachers have focused their part-whole instruction on known-fact strategies such as doubles plus and minus one. Doubles part-whole thinking has short-term value in that it helps students solve some of the more challenging basic facts, including  $5+6$ ,  $6+7$ ,  $7+8$ , and  $8+9$ . It has two limitations, though. First, this strategy does not easily translate to other basic fact problems such as  $6+9$  and  $5+8$ . Thus, teachers often attempt to teach their students several strategies to solve the complete range of over-ten basic fact problems, including doubles strategies, the nines rule, two-aparts, the eights rule, ... With so many strategies and "tricks" to remember, many students get overwhelmed and continue to rely on counting to solve their facts.

A second limitation to focusing on known-fact strategies is that they have limited long-term value in solving multi-digit problems mentally (e.g.,  $27+9$  or  $35+18$ ). Very few multi-digit problems will fall into the doubles-plus-one or two-aparts categories.

A more powerful part-whole strategy for addition is making tens. This is the strategy many Asian students learn for over-ten facts. Once students have memorized their combinations of 10 ( $1+9$ ,  $2+8$ ,  $3+7$ ,  $4+6$ ,  $5+5$ ), they are ready to make tens.

$$\begin{array}{c} 9 + 6 \\ \swarrow \quad \searrow \\ 9 + 1 + 5 \\ \swarrow \quad \searrow \\ 10 + 5 = 15 \end{array}$$

and

$$\begin{array}{c} 8 + 5 \\ \swarrow \quad \searrow \\ 8 + 2 + 3 \\ \swarrow \quad \searrow \\ 10 + 3 = 13 \end{array}$$

## Goals 5 through 9: Subtraction

Most students find subtraction even more challenging than addition. Perhaps because of this, many instructional programs wait until students have achieved success with all of the addition facts before tackling the subtraction facts. For many students, this means they actually have less time during the school year to learn about the facts that are more difficult for them.

This program takes a different approach. Students start with a small chunk of addition facts (1+3, 2+2, 1+4, 2+3). Once they have memorized these facts, they are immediately asked to begin memorizing the related subtraction facts. For each new goal, this same approach provides students with a stable addition foundation on which to develop subtraction confidence.

Once students have completed memorizing the facts through Goal 4, they have the tools to use a part-whole subtraction strategy to solve larger subtraction fact problems. By subtracting from 10 first, students can advance beyond the cognitively demanding method of counting back to solve larger subtraction facts.

### Subtracting from 10 First

$$\begin{array}{ccc} 15 - 8 & \text{or} & 15 \rightarrow 10 + 5 \\ \swarrow \quad \searrow & & \begin{array}{r} 15 \\ - 8 \\ \hline \end{array} \\ 5 + 10 - 8 & & \begin{array}{r} 10 + 5 \\ - 8 \\ \hline 2 + 5 = 7 \end{array} \\ \swarrow \quad \searrow & & \\ 5 + 2 = 7 & & \end{array}$$

Try some for yourself, using both of the recording methods above:

1)  $12 - 9$

2)  $13 - 4$

3)  $11 - 7$

4)  $15 - 6$

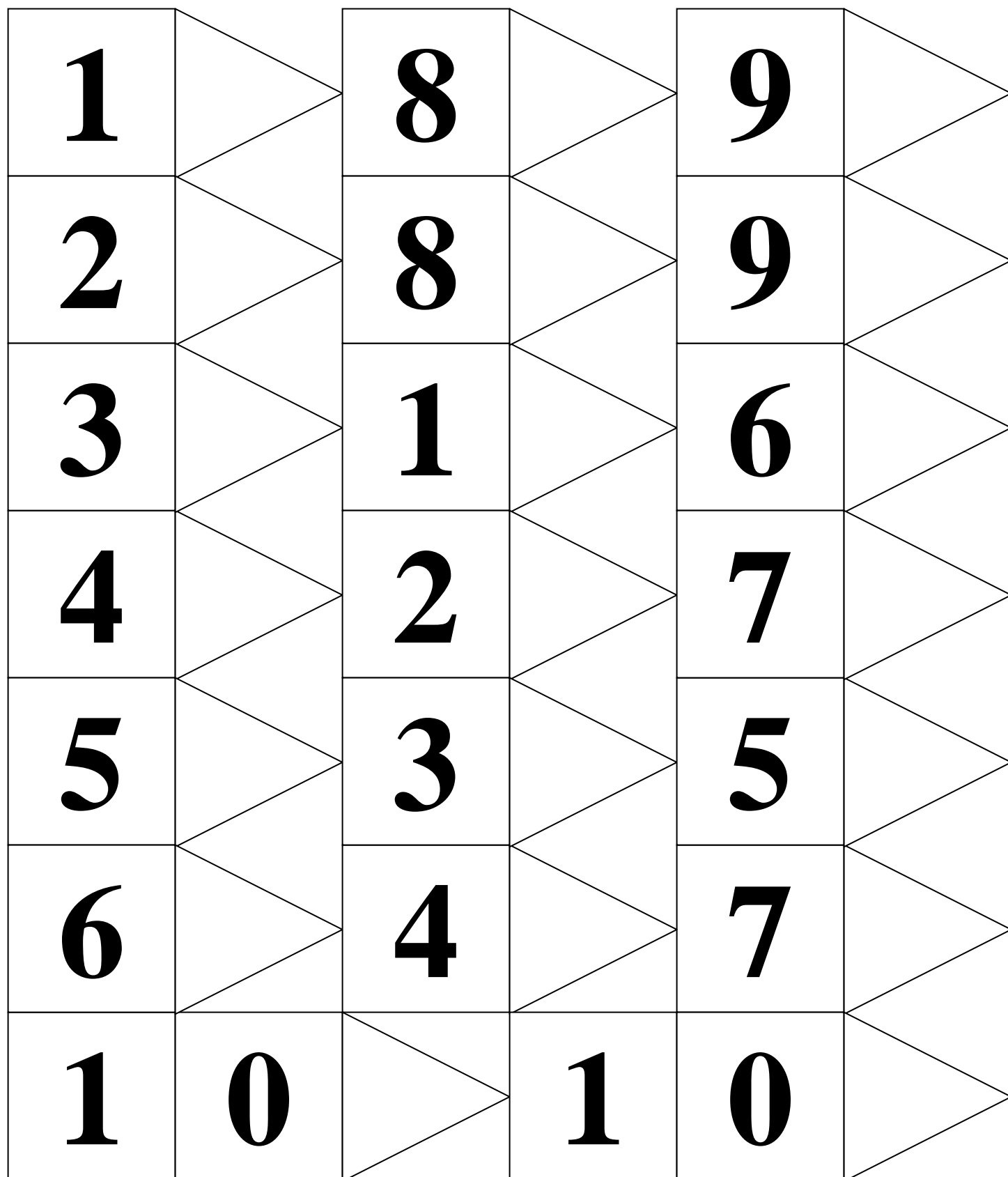
### **A Special Note about Doubles**

Because doubles seem to be more easily memorized than other facts, many teachers have placed an early emphasis on doubles and doubles plus/minus one and two strategies in their classrooms. You may have noticed that doubles are not addressed until Goal 6 in this program. This is primarily because we want to provide students with the tens tools early on, to maximize the extremely powerful tens strategies for both addition and subtraction. When students get to the doubles in Goal 6, they are often pleased to notice how quickly they progress through this goal - at least for addition. It is quite striking how many students who find the addition doubles facts relatively easy need time to make the connections to the doubles subtraction facts.



## Arrow Cards

(See [http://nzmaths.co.nz/Numeracy/Animations/arrow\\_cards.swf](http://nzmaths.co.nz/Numeracy/Animations/arrow_cards.swf) for arrow card information.)



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

# BASIC FACTS WEEKLY PLANNER

Week of \_\_\_\_\_

	Concrete	Representational	Abstract
Monday			
Tuesday			
Wednesday			
Thursday			
Friday			

# BASIC FACTS WEEKLY PLANNER

Week of \_\_\_\_\_

	Concrete	Representational	Abstract
Monday			
Tuesday			
Wednesday			
Thursday			
Friday			